

# Foundations of Linear Programming

Deterministic Models and Core Mathematical Structures for Optimization

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Level: Undergraduate / Bachelor of Science

## Course Overview

This foundational course introduces the mathematical structures, geometric properties, and algorithmic principles of deterministic linear optimization. The curriculum provides undergraduate students with the rigorous analytical tools required to formulate, solve, and analyze large-scale linear models, establishing the essential theoretical baseline for advanced stochastic calculations and heuristic design.

## Core Methodological Modules

### 1. Formulation of Deterministic Models

Mathematical abstraction of real-world allocation and logistics problems. Structural definition of decision variables, linear objective functions, and convex polyhedral constraint spaces. Analysis of deterministic assumptions in resource management.

### 2. Geometry of Linear Programs & Simplex Method

Convexity theory, extreme points, and basic feasible solutions. Algorithmic mechanics of the Primal Simplex method. Geometric intuition of polyhedral vertices and algebraic tracking of basis transformations.

### 3. Duality Theory & Sensitivity Analysis

Mathematical derivation of the Dual problem. Weak and Strong Duality theorems, Complementary Slackness conditions, and economic interpretation of shadow prices. Post-optimal sensitivity analysis of objective coefficients and right-hand side vectors.

### 4. Network Optimization Models

Graph theory foundations for linear networks. Mathematical formulations for shortest path, maximum flow, minimum-cost flow, and transportation problems. Analysis of total unimodularity and its integer implications.

### 5. Discrete Optimization Elements

Introduction to Integer Linear Programming (ILP) and Mixed-Integer Linear Programming (MILP). Structural limitations of linear relaxations. Foundations of exact solution paradigms: Branch-and-Bound and cutting plane concepts.